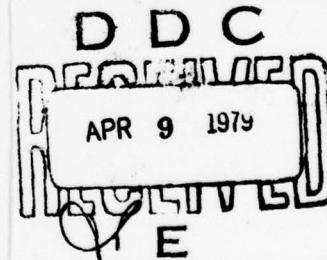MICROCOPY RESOLUTION TEST CHART

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FOREIGN TECHNOLOGY DIVISION

CONVECTION IN A CLOSED CAVITY HEATED FROM THE SIDE
WITH THE DEPENDENCE OF VISCOSITY ON TEMPERATURE

By

Ye. L. Tarunin and V. I. Chernatynskiy

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EDITED TRANSLATION

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

i

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CONVECTION IN A CLOSED CAVITY HEATED FROM THE SIDE WITH THE
DEPENDENCE OF VISCOSITY ON TEMPERATURE

Ye. L. Tarunin and V. I. Chernatynskiy

During the theoretical investigation of free thermal convection,
it is usually assumed that the parameters which characterize the
physical properties of the fluid are constants. Actually, however,
these values depend on temperature and pressure. In many cases, the
dependence of the viscosity coefficient on temperature is especially
significant. High-viscosity fluids - glycerin, oils, certain
petroleum products, and silicone fluids - exhibit a strong dependence
of viscosity on temperature. Even for water, in the temperature range
from 10°C to 100°C the kinematic viscosity coefficient undergoes more
than a four-fold change.

The investigation of the dependence of viscosity on temperature entails considerable mathematical difficulties; therefore, theoretical investigations of convection with consideration of this dependence are rare [1-6]. The precise solution of the problem of stable convection in an infinite vertical flat layer was obtained in [1]. The stability of flows originating in a horizontal flat layer heated from below was considered in [2, 3]. Report [4] considered movement in a convective cell with free boundaries as a model of movement in the Earth's upper mantle. The problem of convection in a vertical cylindrical container was solved in [5]. The method of nets was used to consider the problem of convection in a perfect gas in [6].

This report gives the results of the numerical study of convection in horizontal cylinders with square and round cross sections heated from the side.

1. Formulation of Problem. Main Equations.

When viscosity varies, in vector form the equations of free convection are

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\,\vec{v} = -\frac{1}{\rho_0}\,\nabla p + \nu\Delta\vec{v} + 2(\nabla\nu\nabla)\,\vec{v} +$$

$$+ (\nabla\nu \times \operatorname{rot}\vec{v}) + g\beta T\vec{k}, \qquad (1.1)$$

$$\frac{\partial T}{\partial t} + \vec{v}\,\nabla T = \chi\Delta T, \qquad (1.2)$$

$$\operatorname{div}\vec{v} = 0. \qquad (1.3)$$

Here $\vec{v}$ - velocity, T - temperature, p - pressure, ρ_0 - the mean density of the fluid, g - the acceleration of the force of gravity, β and χ - the coefficients of thermal expansion and heat conductivity, and $\vec{k}$ - the unit vector, directed upward.

Coefficients β and χ are considered to be constant in equations (1.1)-(1.2), while the kinematic viscosity coefficient ν is assumed to be known from the temperature function. The linear dependence of viscosity on temperature is considered in this study:

$$\nu = \nu_0(1 - \alpha T), \qquad (1.4)$$

where ν_0 is the mean viscosity, which corresponds to the mean temperature taken as the computing origin.

Two problems are considered in this study: a) convection in an infinite horizontal cylinder with a square cross section; b) convection in a cylinder with a round cross section.

We will formulate the problem of plane convective movement for a

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\,\vec{v} = -\frac{1}{\rho_0}\,\nabla p + \nu\Delta\vec{v} + 2(\nabla\nu)\,\vec{v} +$$

$$+ (\nabla\nu \times \operatorname{rot} \vec{v}) + g\beta T\vec{k}, \qquad (1.1)$$

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where ν_0 is the mean viscosity, which corresponds to the mean temperature taken as the computing origin.

Two problems are considered in this study: a) convection in an infinite horizontal cylinder with a square cross section; b) convection in a cylinder with a round cross section.

We will formulate the problem of plane convective movement for a

cavity with a square cross section with side d (Fig. 1). All of the boundaries of the region are assumed to be solid. Constant temperatures of $\pm\theta/2$ are maintained on the vertical boundaries, while the horizontal boundaries are assumed to be thermally insulated.

Fig. 1.

a)   b)
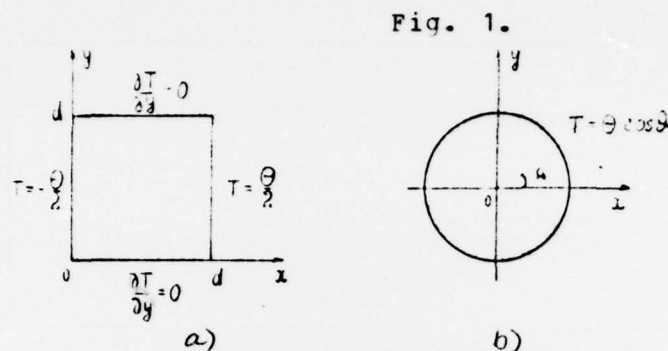
Introducing the current function

$$v_x = \frac{\partial \psi}{\partial y}, \quad v_y = -\frac{\partial \psi}{\partial x}, \qquad (1.5)$$

we will write the equations of free convection in dimensionless form:

$$\frac{\partial \Delta \psi}{\partial t} + \frac{\partial \psi}{\partial y} \cdot \frac{\partial \Delta \psi}{\partial x} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial \Delta \psi}{\partial y} = \Delta \Delta \psi - \gamma \left[T \Delta \Delta \psi + \right.$$

$$+ 2 \frac{\partial T}{\partial x} \cdot \frac{\partial \Delta \psi}{\partial x} + 2 \frac{\partial T}{\partial y} \quad \frac{\partial \Delta \psi}{\partial y} + 4 \frac{\partial^2 T}{\partial x \partial y} \cdot \frac{\partial^2 \psi}{\partial x \partial y} +$$

$$\left. + \left(\frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 T}{\partial y^2} \right) \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right) \right] - G \frac{\partial T}{\partial x}, \qquad (1.6)$$

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \cdot \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial T}{\partial y} = \frac{1}{P} \Delta T. \qquad (1.7)$$

We selected d, d^2/ν_0, ν_0, θ, respectively, as the units of distance,
time, the current function and temperature.

The equations contain three dimensionless parameters - the
Grashof and Prandtl numbers G and P, and also the viscosity
heterogeneity parameter γ:

$$G = \frac{g\beta\Theta d^3}{\nu_0^2}, \quad P = \frac{\nu_0}{\chi}, \quad \gamma = \varkappa\Theta.$$
<div align="right">(1.8)</div>

Parameter γ is related to the value of the viscosity drop p by the ratio

$$p = \frac{\nu_{max}}{\nu_{min}} = \frac{2 + \gamma}{2 - \gamma}.$$
<div align="right">(1.9)</div>

The boundary conditions are:

at $x = 0$: $\psi = \frac{\partial\psi}{\partial x} = 0, \quad T = -\frac{1}{2}$;

at $x = 1$: $\psi = \frac{\partial\psi}{\partial x} = 0, \quad T = +\frac{1}{2}$;   (1.10)

at $y = 0$, $y = 1$: $\psi = \frac{\partial\psi}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = 0$.

This problem was solved by the finite differences method; an explicit system with the substitution of the central differences for the spatial derivatives was used. The procedure of the solution was described earlier [7, 9]. The time interval varied, calculated from the formula:

$$\Delta t = \frac{h^2}{(8 + \psi_m)},$$
<div align="right">(1.11)</div>

where ψ_m is the maximum absolute value of the current functions at the nodes of the network.

The computations were conducted on an Aragats computer on 16×16 and 21×21 networks. The values of the Prandtl number were P = 1 and P = 5. The viscosity heterogeneity parameters selected were γ = 0, 0.5, C.75, 1.0, 1.25 and 1.637 (the last value corresponds to a ten-fold change in viscosity: p = 10). The maximum value of the Grashof number is G = 50·10³.

Now we will consider the formulation of the problem for a round horizontal cylinder with radius R. The boundaries are heated according to the cosine law, sc that the maximum temperature θ corresponds to angle ϑ = 0°, and the minimum θ - to the value ϑ = 180°. In cylindrical coordinates and dimensionless form, the equations of plane convective movement are:

$$\frac{\partial \varphi}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi}{\partial r} \cdot \frac{\partial \varphi}{\partial \vartheta} - \frac{\partial \psi}{\partial \vartheta} \cdot \frac{\partial \varphi}{\partial r}\right) = \Delta \varphi + G\left(\frac{\partial T}{\partial \vartheta}\frac{\sin \vartheta}{r} - \frac{\partial T}{\partial r}\cos \vartheta\right) -$$
$$- \gamma\left[T\Delta\varphi + 2\frac{\partial T}{\partial r} \cdot \frac{\partial \varphi}{\partial r} + \frac{2}{r^2}\frac{\partial T}{\partial \vartheta} \cdot \frac{\partial \varphi}{\partial \vartheta} - \left(\frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r}\frac{\partial \psi}{\partial r} - \right.\right.$$
$$\left.- \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \vartheta^2}\right)\left(\frac{\partial^2 T}{\partial r^2} - \frac{1}{r}\frac{\partial T}{\partial r} - \frac{1}{r^2}\frac{\partial^2 T}{\partial \vartheta^2}\right) -$$
$$\left.- \frac{4}{r^2}\left(\frac{\partial^2 \psi}{\partial r \partial \vartheta} - \frac{1}{r}\frac{\partial \psi}{\partial \vartheta}\right)\left(\frac{\partial^2 T}{\partial r \partial \vartheta} - \frac{1}{r}\frac{\partial T}{\partial \vartheta}\right)\right]. \qquad (1.12)$$
$$-\Delta \psi = \varphi, \qquad (1.13)$$
$$\frac{\partial T}{\partial t} + \frac{1}{r}\left(\frac{\partial \psi}{\partial r} \cdot \frac{\partial T}{\partial \vartheta} - \frac{\partial \psi}{\partial \vartheta} \cdot \frac{\partial T}{\partial r}\right) = \frac{1}{P} \Delta T. \qquad (1.14)$$

Here the Prandtl and Grashof numbers were determined from the mean viscosity ν_0:

$$P = \frac{\nu_0}{\chi}, \quad G = \frac{g\beta\theta R^3}{\nu_0^2}. \qquad (1.15)$$

The current function was introduced in the usual manner:

$$v_r = -\frac{1}{r}\frac{\partial \psi}{\partial \vartheta}, \quad v_\vartheta = \frac{\partial \psi}{\partial r}. \qquad (1.16)$$

R, R^2/ν_0, ν_0 and θ are the units of distance, time, current function, and temperature. It should be pointed out that the total temperature difference in this problem is 2θ, which causes the sense of parameter γ to change somewhat; the value $\gamma = 0.818$ now corresponds to a ten-fold drop in viscosity.

The boundary conditions of the problem are:

$$\text{at } r = 1: \ \psi = 0, \ \frac{\partial \psi}{\partial r} = 0, \ T = \cos \vartheta. \qquad (1.17)$$

The method of nets was used to solve problems (1.12)-(1.17). A network which was uniform with respect to the radius and angle with ten steps on the radius and 16 steps on the angle was used in the main calculations. A network which was irregular over the radius (thickening at the edge) was used in some of the calculations. The order of approximation of the equations and boundary conditions was equal to $0(h^2)$, where h is the maximum distance between the nodes of the network. Symmetrical differences were used to replace the spatial derivatives. The indeterminancy of certain expressions in the differential equations at $r \rightarrow 0$ was revealed by the l'Hôpital rule (see [8], where the procedure for the solution is also described). The time interval was selected with consideration of stability, being

equal to:

$$\Delta t = \frac{\Delta r^2}{4 \left(1 + \frac{1}{\Delta \eta^2} + \max|\tau^2|\right)}. \qquad (1.18)$$

The three-point formula for finding the line perpendicular to the boundary of the temperature gradient was used to calculate the thermal flux.

The calculations were made for the following values of the parameters. The value of the Prandtl number was fixed: $P = 5$; the value of the Grashof number varied up to $G = 15 \cdot 10^3$; the heterogeneity parameter was assigned two values: $\gamma = 0$ and $\gamma = 0.825$.

2. Results of Calculations.

The main results are shown in Figures 1-12 and Tables 1a and 1b, where the integral characteristics of convection are given for the calculated values of the parameters. All of the integral characteristics of the convective process were considered under stable conditions. Stable convection conditions were achieved by the transitional process of attenuating oscillations analogous to the case of homogeneous viscosity [9].

Figures 2-3 give the dependences (the Prandtl number $P = 5$) of

the intensity of convective movement Ψ_m and the dimensionless heat

flux $\overline{N}$ on the Grashof number when the viscosity is constant ($\gamma = 0$)

and there is a ten-fold drop ($\gamma = 1.637$ for the square and $\gamma = 0.825$

for the cylinder). Figures 2a and 3a were plotted for a cavity with a

square cross section, and 2b and 3b - for a round cylinder. As we can

see, the intensity of movement and heat transfer increase somewhat as
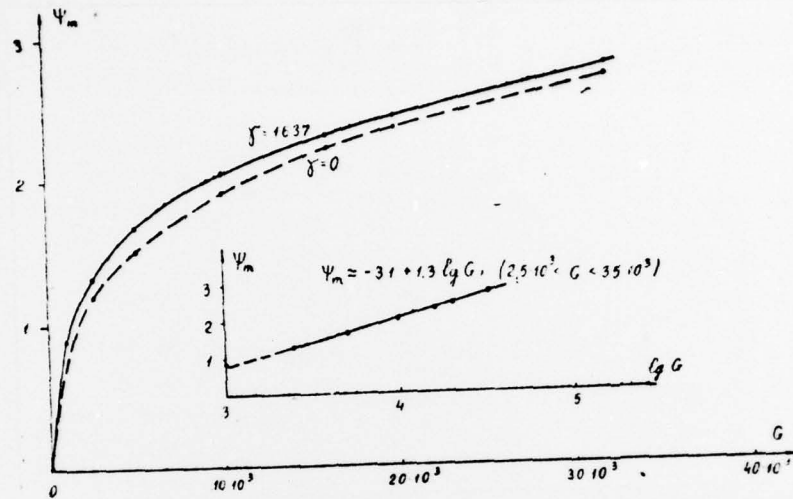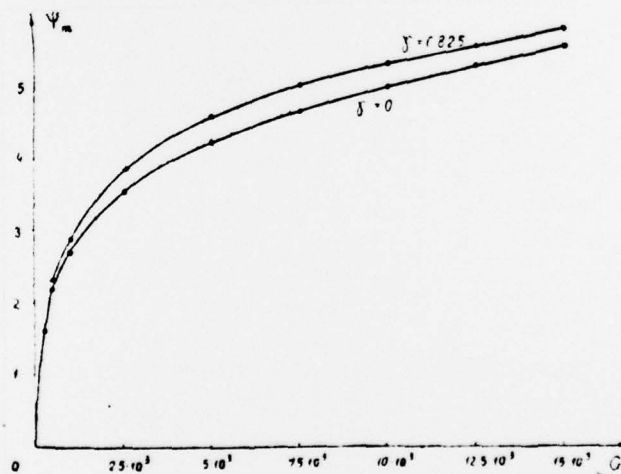
a result of the heterogeneity of the viscosity.
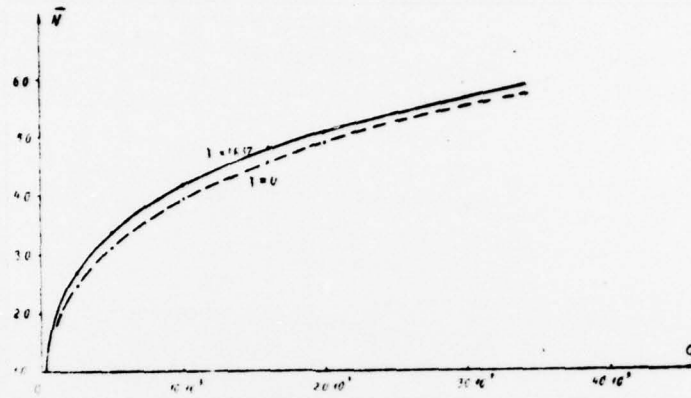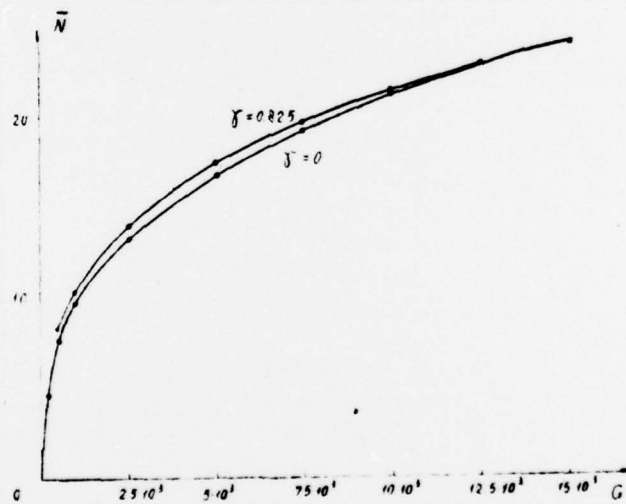
Fig. 2a

Fig. 2b.

Fig. 3a.

Fig. 3b.

It should be noted that due to the asymmetry of the temperature field, the precision of the computation of the heat fluxes N_+ and N_- turns out to be different at $\gamma \neq 0$ (see Tables 1a and 1b). Figure 2 shows the value of N depending on G, equal to $(N_+ + N_-)/2$. The convergence of dependences $\overline{N}(G)$ at large Grashof numbers obviously indicates an increase in the error of the difference system compared to the case $\gamma = 0$, when the number of nonlinear terms in the equation of movement is much smaller.

Table 1a. KEY: (1) Netwcrk.

P	G	γ	ψ_m	v_m	N_+	N_-	$\bar{N}$	Сетка (1)
1	20 000	0	6.84	31.07	2.862	2.862	2.862	15×15
	.	1.0	7.043	33.28	2.919	2.940	2.93	.
	.	0.5	6.91	31.95	2.875	2.881	2.878	.
	.	2.0	7.808	55.8	3.139	3.259	3.22	.
	50 000	0.5	8.96	52.69	3.838	3.931	3.88	.
	.	1.0	9.12	59.5	3.925	3.953	3.94	.
	.	2.0	9.78	81.3	4.092	4.304	4.20	.
5	20 000	1.637	2.429	20.35	4.980	5.224	5.10	15×15
	10 000	.	2.031	14.48	4.068	4.285	4.18	.
	5 000	.	1.66	10.1	3.298	3.445	3.37	.
	2 500	.	1.32	6.7	2.674	2.708	2.69	.
	1 000	.	0.89	3.8	1.969	1.955	1.96	.
	16 000	.	2.30	18.3	4.65	4.94	4.80	.
	32 000	.	2.78	25.4	5 67	5.95	5.81	.
	10 000	.	2.01	14.1	3.99	4.34	4.17	20×20
	20 000	1.5	2.42	19.7	4.982	5.155	5.07	15×15
	.	1.25	2.405	18.8	4.973	5.066	5.02	.
	.	1.00	2.39	17.9	4.953	5.011	4.98	.
	.	0.75	2.38	17.15	4.938	4.971	4.95	.
	.	0.50	2.37	16.5	4.929	4.941	4.935	.

Table 1b.

G	ψ_m		N $(\gamma = 0.825)$		$\bar{N}$	
	$\gamma = 0$	$\gamma = 0.825$	N_+	N_-	$\gamma = 0$	$\gamma = 0.825$
500	2.15	2.31	8.66	7.85	7.64	8.25
1000	2.68	2.87	10.85	9.84	9.61	10.34
2500	3.54	3.85	14.79	13.02	13.1	13.9
5000	4.21	4.58	18.23	16.37	16.75	17.5
7500	4.68	5.04	20.92	18.38	19.28	19.65
10 000	5.02	5.33	22.93	20.02	21.29	21.48
12 500	5.31	5.56	24.26	21.57	22.95	22.92
15 000	5.58	5.85	25.74	22.63	24.28	24.18

The increase in the intensity of movement when viscosity depends on temperature is illustrated in Fig. 4, in which the maximum velocity v_m is shown in dependence on the heterogeneity parameter γ in the case of a region with a square cross section. In the range of γ in question, this dependence ($G = 20 \cdot 10^3$, $P = 1$ and $P = 5$) is close linear.

Fig. 4.
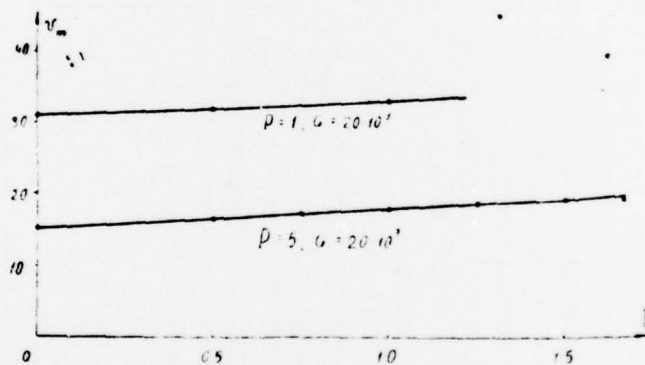
We will point out that the integral characteristics of the convective process vary less with the increase in γ than the local characteristics. In the case of a region with a square cross section, by processing the results for $P = 5$ and $G = 20 \cdot 10^3$ we obtain the following approximate dependences $(0 \leqslant \gamma \leqslant 1.6)$:

$$\overline{N}(\gamma) = \overline{N}(0)(1 + 0.02\gamma),$$
$$\psi_m(\gamma) = \psi_m(0)(1 + 0.02\gamma). \qquad (2.1)$$

Figures 5-12 illustrate the structure of movement and the temperature field at different values of the parameters. As we can see, at sufficiently large values of the Rayleigh number $Ra = G \cdot P$, which corresponds to Figures 5-6 and Figures 11-12, the velocity and temperature boundary layers are well-developed in the flow. Naturally, the effect of the heterogeneity of viscosity increases with the increase in parameter γ. This effect is evident, first, in the fact that flow symmetry increases at $\gamma = 0$. The flow rates on the hot wall increase, and asymmetry of fields ψ and T originates. This asymmetry is evident, in particular, from the shift in the isotherms, the current lines and the vortex centers. The isotherm corresponding to the mean temperature $(T = 0)$ shifts toward the cold fluid.

We will point out that the patterns of movement shown in Figures 9-12 qualitatively agree with the patterns given in experimental study [10].
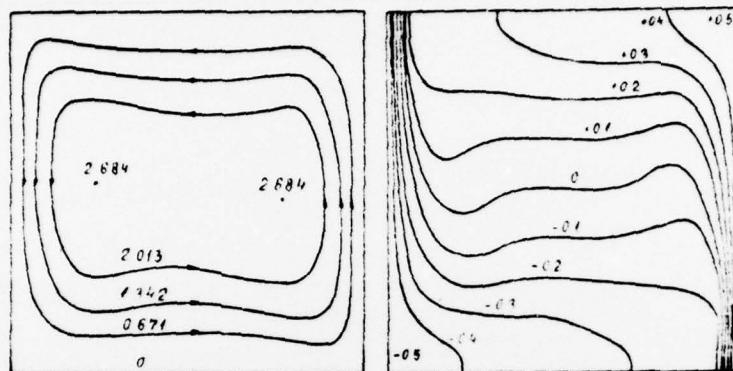
Fig. 5. $G=32 \cdot 10^3$; $P=5$; $\gamma=0$

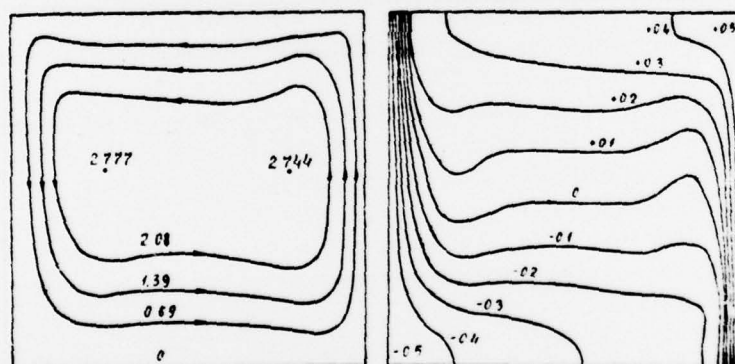Fig. 6. $G=32 \cdot 10^3$; $P=5$; $\gamma=1.637$

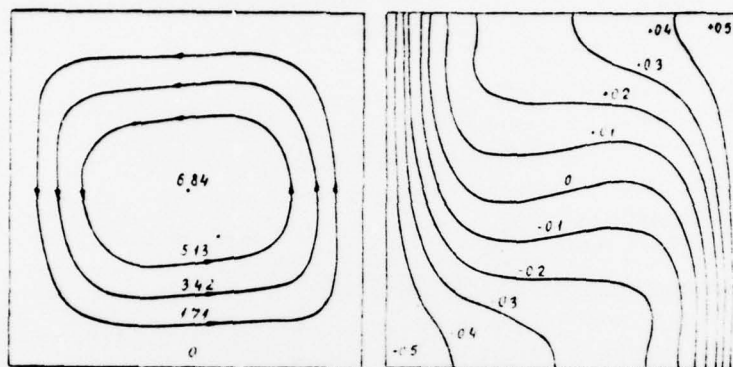Fig. 7. $G = 20 \cdot 10^3$; $P = 1$; $\gamma = 0$
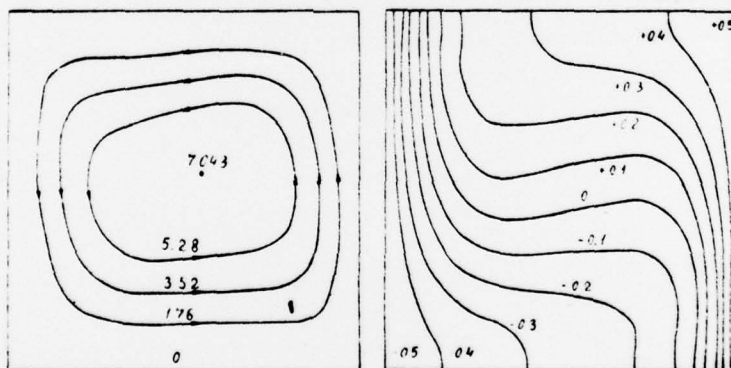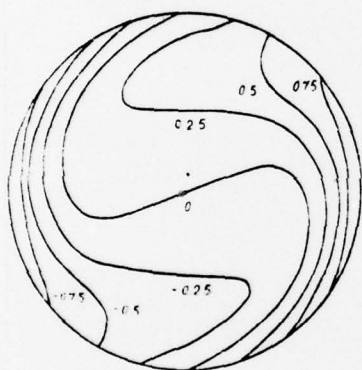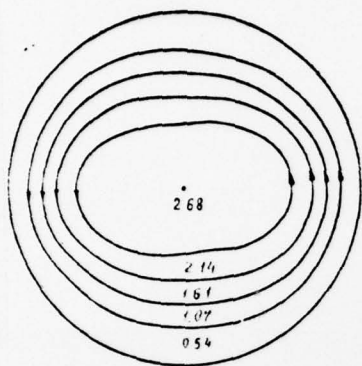

Fig. 8. $G = 20 \cdot 10^3$; $P = 1$; $\gamma = 1$

Fig. 9. $G=10^3$; $P=5$; $\gamma=0$ Fig. 10. $G=10^3$; $P=5$; $\gamma=0.825$

2.68

2.14
1.61
1.07
0.54

2.87

2.29
1.72
1.15
0.57

0.5 0.75
0.25
0

-0.25
-0.75 -0.5

0.5 0.75
0.25
0

-0.75 -0.5 -0.25
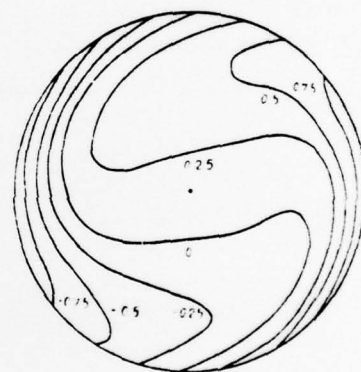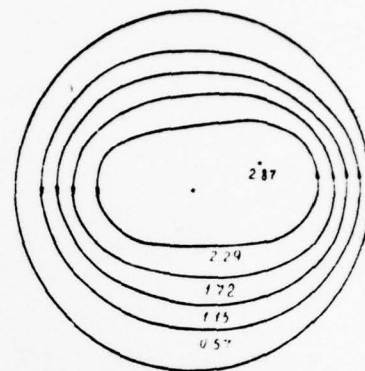
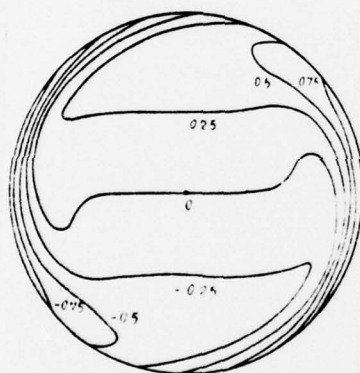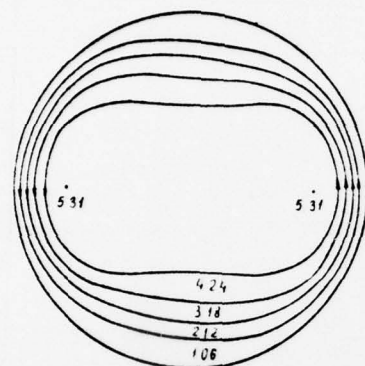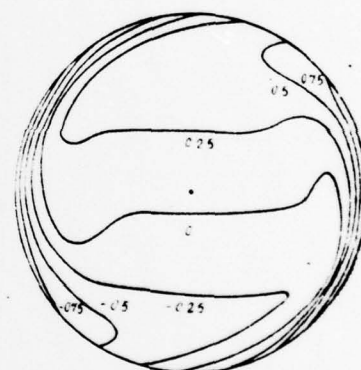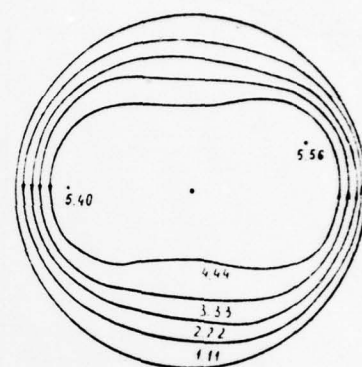Fig. 11. $G=125 \cdot 10^3$; $P=5$; $\gamma=0$

Fig. 12. $G=125 \cdot 10^3$; $P=5$; $\gamma=0.825$

Thus, the results of the calculations indicate that the dependence of viscosity on temperature causes a more or less significant change in the local characteristics of convective flow. As for the integral characteristics - the maximum value of the current function and the dimensionless heat flux - being determined by the mean viscosity, they change with consideration of the temperature dependence of viscosity in relatively narrow limits.

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